24.01.19 Volker.Freudenthaler@lmu.de
Improvements_of_lidar_correction_ghk_ver.0.9.8_190124

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Note: Equation references Eq. (*) are with respect to the paper
About the effects of polarising optics on lidar signals and the Δ90 calibration
http://www.atmos-meas-tech.net/9/4181/2016/amt-9-4181-2016.html

Correction of the Python script ver. 0.9.6 => ver. 0.9.8

A bug has been found in the Python script lidar_correction_ghk.py, which effects the values of the GHK-parameters, but not the correct retrieval of the linear depolaristion ratio δ with them, because the individual errors of the GHK-parameters compensate each other in Eqs. (1) to (3) below [Eqs. (62*), (60*), and (83*)ff in the paper].

Buggy version: *lidar_correction_ghk_0.9.6.py,*
published as *lidar_correction_ghk.py* with tag 0.9.6
on 2017-11-15 under https://bitbucket.org/iannis_b/atmospheric_lidar_ghk

Corrected version: *lidar_correction_ghk_0.9.8d_Py3.7.py*
published as *lidar_correction_ghk.py* with tag 0.9.8
on 2019-01-22 under https://bitbucket.org/iannis_b/atmospheric_lidar_ghk
(tested with Python 3.7)

$$\delta = \frac{\delta^*(G_T + H_T) - (G_R + H_R)}{(G_R - H_R) - \delta^*(G_T - H_T)} \tag{1}$$

$$\delta^* = \frac{1}{\eta}\frac{I_R}{I_T} = \frac{K}{\eta^*}\frac{I_R}{I_T}(0°) \tag{2}$$

$$K = \frac{\eta^*}{\eta} = \frac{\eta_T T_T}{\eta_R T_R}\eta^* = \frac{\eta_T T_T}{\eta_R T_R}\frac{I_R}{I_T}(\pm 45°) \tag{3}$$

Effects of the bug:
G and *H* about a factor of two larger.

$K \neq 1$ for cases where $K = \eta^*/\eta$ should be = 1 according to the paper.
No effects on the correction of the measured η^* and δ^* to yield δ .
No effects on the error calculation of δ in the script.

What should you do?

As the correction of η^* and δ^* works fine with the "wrong" GHK-parameters ver. 0.9.6, you can continue using them with the SCC. But you should keep in mind, that the GHK-values are not same when deriving them directly from the equations of the paper. Maybe its better to derive a new set of GHK-parameters with the corrected script and use them in the future. And maybe someone can compare a SCC retrieval with the wrong and with the correct GHKs to confirm that both are the same, and please publish the confirmation in the EARLINET forum.

Detailed explanation and proof that the "wrong" G'H'K' still work correct

The term

$T_{AS\phi} = 1 + c_{2\phi} D_A D_S$

must be included in the unpolarized transmittance of the cleaned analyser $T_S^{\#}$ (Eq.(9)) and not in the GH-parameters as in Eq.(8).

From Eq. (S.10.10.1*) we get for the cleaned analyser with rotated linear polariser M_A

$\gamma = 0 \Rightarrow$

$$\frac{\langle \mathbf{M}_A(\phi)\mathbf{M}_S(0)|}{T_A T_S} = \langle 1 + c_{2\phi} D_A D_S \quad D_S + c_{2\phi} D_A \quad s_{2\phi} D_A Z_S c_S \quad s_{2\phi} D_A Z_S s_S | \tag{4}$$

and considering Eq. (D.5*) for the 0° or 90° orientation of the polarising beam splitter cube with

$$\mathbf{R}_y = \mathbf{R}(y) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & y & 0 & 0 \\ 0 & 0 & y & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \text{ Eq. (47*)} \tag{5}$$

gives

$$\frac{\langle \mathbf{A}_S|}{T_A T_S} = \frac{\langle \mathbf{M}_A(\phi)\mathbf{M}_S(0)\mathbf{R}_y|}{T_A T_S} = \langle 1 + c_{2\phi} D_A D_S \quad D_S + c_{2\phi} D_A \quad s_{2\phi} D_A Z_S c_S \quad s_{2\phi} D_A Z_S s_S | \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & y & 0 & 0 \\ 0 & 0 & y & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \tag{6}$$

$$= \langle 1 + c_{2\phi} D_A D_S \quad y(D_S + c_{2\phi} D_A) \quad y s_{2\phi} D_A Z_S c_S \quad s_{2\phi} D_A Z_S s_S |$$

and the normalised cleaned analyser vector

Equation references Eq. (*) are with respect to the paper "*About the effects of polarising optics on lidar signals and the Δ90 calibration*"
http://www.atmos-meas-tech.net/9/4181/2016/amt-9-4181-2016.html

$$\frac{\langle \mathbf{A}_S|}{T_A T_S\left(1+c_{2\phi}D_A D_S\right)} = \frac{\langle \mathbf{A}_S|}{T_S^{\#}} = \frac{\langle \mathbf{A}_S|}{T_A T_S T_{AS\phi}} = \left\langle 1 \quad \frac{y\left(D_S + c_{2\phi}D_A\right)}{1+c_{2\phi}D_A D_S} \quad \frac{ys_{2\phi}D_A Z_S c_S}{1+c_{2\phi}D_A D_S} \quad \frac{s_{2\phi}D_A Z_S s_S}{1+c_{2\phi}D_A D_S}\right| =$$

$$= \left\langle 1 \quad \frac{y\left(D_S + c_{2\phi}D_A\right)}{T_{AS\phi}} \quad \frac{ys_{2\phi}D_A Z_S c_S}{T_{AS\phi}} \quad \frac{s_{2\phi}D_A Z_S s_S}{T_{AS\phi}}\right| \qquad (7)$$

with the unpolarised transmittance

$$T_S^{\#} = T_A T_S\left(1+c_{2\phi}D_A D_S\right) = T_A T_S T_{AS\phi}$$

This had not been considered consistently until now.
It is **corrected** in **lidar_correction_ghk_0.9.8_Py3.7.py**
(and in MML_v09_RotCal_A-Lidar_compare_with_python_code.ods)

The script calculates/simulates six signals: four calibration signals ($I_R(\pm 45°)$, $I_T(\pm 45°)$) and two standard 0°-signals ($I_R(0°)$, $I_T(0°)$).
In the old script version 0.9.6 Eq. (8) had been used and the terms $T_{AS\Phi}$ were included in the GH-parameters, let's name them G' and H', and in all six simulated signals.

$$I_S = \eta_S T_A T_S T_O T_{rot} F_{11} T_E I_L \left(G_S{}' + aH_S{}'\right) \quad \text{with} \quad G_S{}' = G_S T_{AS\phi}, H_S{}' = H_S T_{AS\phi} \qquad (8)$$

These signals are the same as derived with the correct equations in vers. 0.9.8:

$$I_S = \eta_S T_S^{\#} T_O T_{rot} F_{11} T_E I_L \left(G_S + aH_S\right) \quad \text{with} \quad T_S^{\#} = T_A T_S T_{AS\phi} \qquad (9)$$

The signal ratio η^* , the calibration factor η, and the correction parameter K' were calculated in vers. 0.9.6 from

$$\eta' = \frac{T_A T_R}{T_A T_T} \quad \text{and} \quad \eta^* = \frac{I_R}{I_T}\left(\pm 45°\right) \Rightarrow K' = \frac{\eta^*}{\eta'} \qquad (10)$$

where green/red color marks the correctly/incorrectly calculated parameters/variables,
and in vers. 0.9.8 from

$$\eta = \frac{T_R^{\#}}{T_T^{\#}} = \frac{T_A T_R T_{AR\phi}}{T_A T_T T_{AT\phi}} \quad \text{and} \quad \eta^* = \frac{I_R}{I_T}\left(\pm 45°\right) \Rightarrow K = \frac{\eta^*}{\eta} \qquad (11)$$

From this we derive

$$\eta = \eta' \frac{T_{AR\phi}}{T_{AT\phi}}, \quad K' = K\frac{T_{AR\phi}}{T_{AT\phi}} \quad \text{and} \quad \delta^{*\prime} = \frac{1}{\eta'}\frac{I_R}{I_T}\left(0°\right) = \frac{K'}{\eta^*}\frac{I_R}{I_T}\left(0°\right) \qquad (12)$$

Correcting η^* and δ^* with the GHK from vers. 0.9.6 with the "wrong" G', H', and K' results in the same as doing it with the correct GHK from vers. 0.9.8: (13)

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http://www.atmos-meas-tech.net/9/4181/2016/amt-9-4181-2016.html

$$\delta = \frac{\delta^{*\prime}\left(G_T{}' + H_T{}'\right) - \left(G_R{}' + H_R{}'\right)}{\left(G_R{}' - H_R{}'\right) - \delta^{*\prime}\left(G_T{}' - H_T{}'\right)} = \frac{\frac{K'}{\eta^*}\frac{I_R}{I_T}(0°)\left(G_T{}' + H_T{}'\right) - \left(G_R{}' + H_R{}'\right)}{\left(G_R{}' - H_R{}'\right) - \frac{K'}{\eta^*}\frac{I_R}{I_T}(0°)\left(G_T{}' - H_T{}'\right)} =$$

$$= \frac{\frac{K}{\eta^*}\frac{T_{AR\phi}}{T_{AT\phi}}\frac{I_R}{I_T}(0°) T_{AT\phi}\left(G_T + H_T\right) - T_{AR\phi}\left(G_R + H_R\right)}{T_{AR\phi}\left(G_R - H_R\right) - \frac{K}{\eta^*}\frac{T_{AR\phi}}{T_{AT\phi}}\frac{I_R}{I_T}(0°) T_{AT\phi}\left(G_T - H_T\right)} = \frac{\frac{K}{\eta^*}\frac{I_R}{I_T}(0°)\left(G_T + H_T\right) - \left(G_R + H_R\right)}{\left(G_R - H_R\right) - \frac{K}{\eta^*}\frac{I_R}{I_T}(0°)\left(G_T - H_T\right)} = \tag{14}$$

$$= \frac{\delta^*\left(G_T + H_T\right) - \left(G_R + H_R\right)}{\left(G_R - H_R\right) - \delta^*\left(G_T - H_T\right)}$$

Equation references Eq. (*) are with respect to the paper "*About the effects of polarising optics on lidar signals and the Δ90 calibration*"

http://www.atmos-meas-tech.net/9/4181/2016/amt-9-4181-2016.html

Additions to the Python script ver. 0.9.8d

The new script version 0.9.8 includes following additions compared to ver. 0.9.6:

Calculation of signal noise errors (experimental)

The errors resulting from signal noise of the calibration and standard measurements can be considered now in a simple way. The standard deviations of the number of temporally and spatially averaged photon counts in each signal, which are considered to be statistically independent, are included in the same way as the systematic uncertainties of the optical elements. This is not the correct way for combing random and systematic errors, but nevertheless gives a first insight in the relative importance of the noise.

Two methods to determine the signal intensities are included.
method 1: The number of photon counts in the parallel signal before the telescope are fixed, and all other signals are retrieved from this considering the individual polarized and unpolarized transmittances including the additional ND-filter.
method 2: The number of photon counts as stored in the data recorder of the parallel signal and of the calibration signals are given in the input_file. The standard deviations are directly retrieved from those with any further attenuations. The numbers of photons in each signal could be taken directly from real measured signals.

The calculation of noise errors is an experimental feature, in development, and disabled by default.
Please contact me if you want to use it.

Additional ND filter attenuation during the calibration

Until now I recommended to attenuate the whole lidar receiver during the Δ90-calibration measurements with a neutral density attenuator (polka-dot attenuator before the telescope) with $T_{ND} \approx 0.1$ in order to prevent the saturation of the cross signals during the Δ90-calibration (**method 1**). But here also the parallel signal is attenuated and becomes very nosiy. Some lidars include therefore an insertable ND-filter only in the cross signal (**method 2**), but here the uncertainty of its transmittance T_{ND} is an additional error source. The additional error sources of the two methods are:
method 1: increase of signal noise in cross and parallel signals
method 2: increase of signal noise in cross signal & systematic uncertainty of T_{ND}
In order to simulate the uncertainties of method 1 and 2 and to compare them, I included following parameters in the script and input_file:

```
# NEW --- Additional ND filter transmission (attenuation) during the
calibration
TCalT, dTCalT, nTCalT  = 1, 0.01, 0    # transmitting path, default 1, 0, 0
TCalR, dTCalR, nTCalR = 0.1, 0.001, 1    # reflecting path, default 1, 0, 0
```

As T_{ND} has an effect on signal noise, the change of the noise errors between the methods should be considered in an overall error calculation. As stated above, this feature is experimental.
But first results indicate that the relative error of T_{ND} in the cross channel in method 2 must be in the order of 0.001 in order to clearly outmatch method 1.

Corrections of the paper

About the effects of polarising optics on lidar signals and the Δ90 calibration
http://www.atmos-meas-tech.net/9/4181/2016/amt-9-4181-2016.html

The paper version *About the effects of polarising optics_4b_1L_corr.pdf*, also downloadable from the repository https://bitbucket.org/iannis_b/atmospheric_lidar_ghk, contains some corrections and includes the supplement https://www.atmos-meas-tech.net/9/4181/2016/amt-9-4181-2016-supplement.pdf.

Corrected: Eqs. (116), (125), (126), (C.1) to (C.13), (S.10.16.1)

Linear polariser calibrator

Eqs. (125), (126)

From Sect. 8.1: Calibration with a linear polariser before the polarising beam-splitter [Eq. (123*)ff]

$$\frac{I_S}{\eta_S T_S T_P I_{in}} = \frac{\langle \mathbf{M}_S \mathbf{R}_y | \mathbf{M}_P (x45° + \varepsilon) | \boldsymbol{I}_{in} \rangle}{T_S T_P I_{in}} = \left\langle \begin{matrix} 1 - xys_{2\varepsilon} D_P D_S \\ -xs_{2\varepsilon} D_P + yD_S \left(1 - c_{2\varepsilon}^2 W_P\right) \\ xc_{2\varepsilon} D_P - ys_{2\varepsilon} c_{2\varepsilon} W_P D_S \\ -xyc_{2\varepsilon} Z_P s_P D_S \end{matrix} \middle\| \begin{matrix} i_{in} \\ q_{in} \\ u_{in} \\ v_{in} \end{matrix} \right\rangle = \quad (15)$$

$$= i_{in} + yD_S \left[q_{in} - c_{2\varepsilon} W_P \left(c_{2\varepsilon} q_{in} + s_{2\varepsilon} u_{in} \right) \right] - x \left[D_P \left(s_{2\varepsilon} q_{in} - c_{2\varepsilon} u_{in} \right) + yD_S \left(s_{2\varepsilon} D_P i_{in} + c_{2\varepsilon} Z_P s_P v_{in} \right) \right]$$

Without calibrator rotation error ε

$\varepsilon = 0 \Rightarrow$

$$\frac{I_S}{\eta_S T_S T_P I_{in}} = i_{in} + yD_S [1 - W_P] q_{in} + x [u_{in} D_P - yD_S Z_P s_P v_{in}] = i_{in} + xD_P u_{in} + yD_S Z_P (c_P q_{in} - s_P v_{in}) \quad (16)$$

Substitution of the linear polariser diattenuation D_P and of Z_P by the lin.polariser extinction ratio ρ by means of Eq. (S.10.10.8*) (c_P = cos(retardation))

$$\rho = \frac{k_2}{k_1} = \frac{T_P^s}{T_P^p} = \frac{1 - D_P}{1 + D_P} \Rightarrow \quad (17)$$

$$D_P = \frac{1 - \rho}{1 + \rho} \approx 1 - 2\rho \quad (18)$$

$$Z_P = \sqrt{1 - D_P^{\,2}} \approx \sqrt{1 - (1 - 2\rho)^2} \approx 2\sqrt{\rho} \quad (19)$$

we get from Eq. (16) with a horizontal-linearly polarised input I_{in}, the correct version of Eq. (125*):

$$K = \frac{\eta^*}{\eta} = \frac{1 + yD_R Z_P c_P}{1 + yD_T Z_P c_P} \approx \frac{1 + yD_R 2\sqrt{\rho} c_P}{1 + yD_T 2\sqrt{\rho} c_P} \quad (20)$$

Eq. (20) with a cleaned analyser gives the correct version of Eq. (126*):

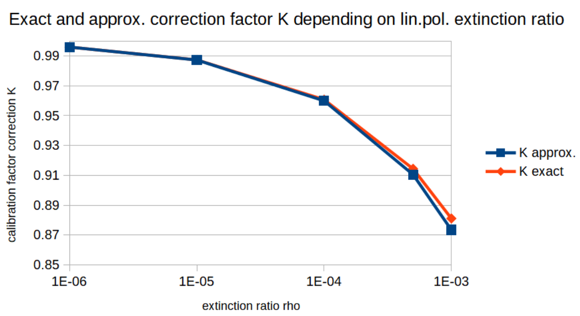
$$K = \frac{\eta^*}{\eta} \approx \frac{1 - 2y\sqrt{\rho}c_P}{1 + 2y\sqrt{\rho}c_P} \approx 1 - 4y\sqrt{\rho}c_P, \quad c_P = \cos(retardation) \tag{21}$$

We see in Eq.(21) that the retardation of the linear polariser calibrator plays an important role.
The corresponding plot below shows
- that an error of the extinction ratio of 1e-5 leads to an error of about 1% in the calibration factor correction K and hence of the calibration factor eta,
- that the change of the retardation from 0° to 180° (=> cos(retardation) c_P from 1 to -1) modulates the full correction term of 4*sqrt(ρ).

This means in case of extinction ratio 1e-4 that for
an uncertainty of c_P between +1 and -1
K could be between 0.96 and 1.04.

And this would lead at δ = 0.5 to an error of +-0.02.
Because K is in first approx. a factor for δ, the absolute error at small δ is also small.

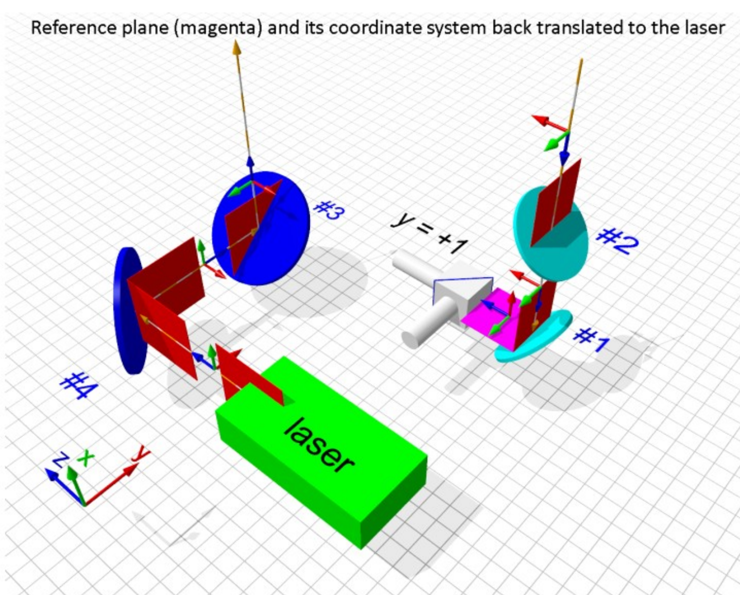
The approximation in Eq.(21) is very good for $\rho \le$ 1e-4 (see).
For c_P = 1 (retardation = 0).



(Error of linear polarizer calibrator.ods)

About the correct rotation of the laser polarisation and of the optical elements

Müller matrices of optical elements and Stokes vectors are all relative to a reference plane.
When I began to develop the equations of my paper, I assumed that all lidar systems have a fixed polaising beams plitter (PBS) cube, whichs incidence plane and its mechanical orientation can be determined quite easily, because it is a cube. A complication arose from the fact that some lidar systems include the possibility to rotate the (PBS) and its attached detectors, and when rotating it, all the Müller matrices of the other optical elements would have to be redefined. The solution was to include in the theory the rotation matrix $\mathbf{R}_y$ with the polarising beam-splitter orientation parameter y [Eq. (47*)]. With this the reference plane for the whole lidar setup is a plane in front of the PBS as shown in fig. 1 as magenta plane behind beamsplitter #1. But because now the reference plane is defined independently from the PBS, it is not obvious how to choose it, and it might be confusing to determine the rotation of the optical elements and of the laser polarization. Figures 1 and 2 below should clearify that.
Firstly it must be noted that Ts, Tp, Rs, and Rp of the transmittances and reflectances of optical elements are defined with respect to their incidence planes, and therefore the Müller matrices determined from Ts, Tp, Rs, and Rp according to Eqs.(14*) and (16*) are also defined with respect to their incidence planes. The red planes in Fig. 1 are the translations of the reference plane to the corresponding optical elements, and only the mirror #3 has the red plane shown in its incidence plane. The Müller matrices of elements #1, #2, and #4 must be rotated by 90° in the description of the lidar setup [Eq.(48*)]. With this translation of the reference plane also the correct rotation of the laser polarisation can be determined. If, for example, the plane of linear polarisation of the laser is horizontal, its rotation α with respect to the reference plane in the Müller-Stokes description of the lidar [Eq.(48*)] is 90°. Figure 2 shows three more examples as demonstration.



We define y = +1 as in the picture above! Then the reference plane is the incidence plane of the PBC (magenta). The red planes are the back-translations of the reference plane to the incidence and excidence planes of the optical elements. The multiplication of Müller matrices is in direction of the propagation of light.

Figure 1

Equation references Eq. (*) are with respect to the paper "*About the effects of polarising optics on lidar signals and the Δ90 calibration*"

http://www.atmos-meas-tech.net/9/4181/2016/amt-9-4181-2016.html

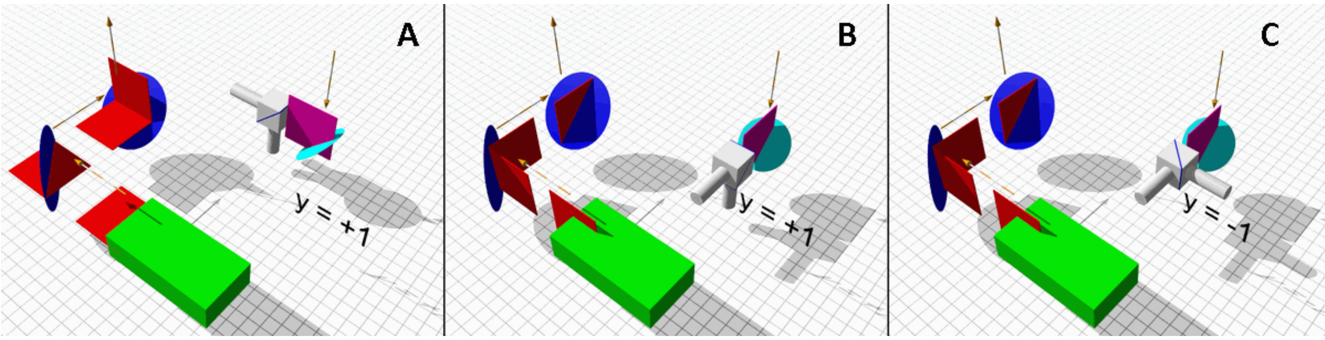


Figure 2